Preprint MPI H-V5-2001

On the flavor mixing by a Light-Cone Hamiltonian and the isotopic spin in QCD

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Abstract

The structure of an effective light-cone Hamiltonian as recently derived is analyzed with emphasis on its prediction for flavor mixing in physical mesons. In a (perhaps over-simplified) model with one adjustable parameter, the empirical masses of all 25 pseudo-scalar mesons which are possible for 5 flavors are reproduced (almost) quantitatively. These results are coupled with explicit numerical estimates of flavor mixing. In the present approach, the well-known mass degeneracy of the pion triplet is caused by the mass degeneracy of the up and down quark.

1 Introduction

In 1932, Heisenberg has postulated isotopic spin as a general symmetry of nature to describe systematically atomic nuclei. Isospin prevails to be important in hadronic physics, both in concept [1] and experiment [2]. But in the fundamental hadronic theory, in the gauge theory of quantum chromodynamics (QCD), isospin symmetry is not manifest in any obvious way. The present work is intended to contribute yet an other facet to an old and immanent question [1], without addressing however to be complete or exclusive.

Nowadays isospin symmetry is believed to be part of the more general chiral symmetry to the extent that the symmetry is thought to be manifest at sufficiently high temperatures and broken after a chiral phase transition in which the quarks acquire mass. These ideas can be modeled by lattice gauge theory (LGT) [3]. But when it comes down to calculate hadrons in their ground state (at absolute zero), LGT has some difficulties to calculate with the same quality of approximations for all the hadrons, because of the enormously different mass scales in the problem. Particularly, it is not easy to extrapolate reliably down to the very light and small mesons like the pions.

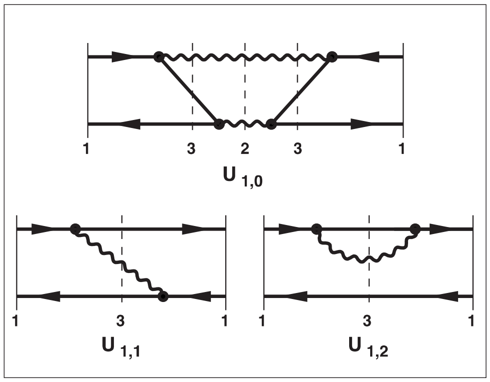

Fig. 1. The three graphs of the effective interaction in the $q\bar{q}$-space. The lower two graphs correspond to the effective one-gluon-exchange interaction U_{OGE}, the upper corresponds to the effective two-gluon-annihilation interaction U_{TGA}. The figure is taken from Ref.[5].

	$u\overline{d}$	$u\overline{s}$	$d\overline{s}$	$d\overline{u}$	$s\overline{u}$	$s\overline{d}$	$u\overline{u}$	$d\overline{d}$	$s\overline{s}$
$u\overline{d}$	E_1	0	0	0	0	0	0	0	0
$u\overline{s}$	0	E_2	0	0	0	0	0	0	0
$d\overline{s}$	0	0	E_3	0	0	0	0	0	0
$d\overline{u}$	0	0	0	E'_1	0	0	0	0	0
$s\overline{u}$	0	0	0	0	E'_2	0	0	0	0
$s\overline{d}$	0	0	0	0	0	E'_3	0	0	0
$u\overline{u}$	0	0	0	0	0	0	e_4	A_5	A_6
$d\overline{d}$	0	0	0	0	0	0	A_5	e_7	A_8
$s\overline{s}$	0	0	0	0	0	0	A_6	A_8	e_9

Fig. 2. The kernel of $H_{\mathrm{LC,eff}}$ displayed as a block matrix illustrates the flavor mixing in QCD. $e_i \equiv E_i + A_i$.

Recently, an equally non-perturbative alternative has gradually emerged as reviewed in [4]. The light-cone approach addresses to diagonalize the (light-cone) Hamiltonian, $H_{LC}|\Psi\rangle = M^2|\Psi\rangle$, and to calculate the spectra and invariant masses (squared) of physical particles. In particular the method addresses to calculate the associated wave functions $\Psi_n = \Psi_{q\bar{q}}, \Psi_{q\bar{q}g}, \ldots$, which are the Fock-space projections of the hadrons eigenstate. The total wave function for a meson is then $|\Psi_{meson}\rangle = \sum_i (\Psi_{q\bar{q}}(x_i, \vec{k}_{\perp_i}, \lambda_i)|q\bar{q}\rangle + \Psi_{q\bar{q}g}(x_i, \vec{k}_{\perp_i}, \lambda_i)|q\bar{q}g\rangle + \ldots)$, for example.

I present in section 2 some general aspects of the method, based on which I formulate in section 3 a sufficiently simple model. In order to be as concrete and pedagogic as possible, I continue in section 4 with an over-simplified model for 2 and 3 flavors which can be solved in closed form. I generalize in section 5 to five flavors, and compare to experiment. In section 6, I draw the conclusions.

2 General considerations

Let me review in short some general aspects of the approach. The full light-cone Hamiltonian for gauge theory with its complicated many-body aspects is reduced in [5] by the method of iterated resolvents to the effective Hamiltonian

$$H_{\mathrm{LC,eff}} = T + U_{\mathrm{OGE}} + U_{\mathrm{TGA}}. \tag{1}$$

By definition it acts only in the Fock space of a single quark and anti-quark. The kinetic energy T be definition is diagonal and is the only part of the effective Hamiltonian surviving in the limit of vanishing coupling constant. Note that it has the dimension of a mass-squared, like the other operators in the (light-cone) Hamiltonian. The interaction (kernel) has three contributions, which are displayed diagrammatically in Fig. 1.

The diagrams in the figure are very compact. They are 'energy' but not Feynman-diagrams; all particle lines and all propagators are 'effective' but well defined and represent summations over all orders [5]. In diagram $U_{1,2}$, the effective gluon is absorbed on the same line and does not change its kinematical state. It therefore generates effective masses (in contrast to the bare Lagrangian ones). A certain part of them can be absorbed in T. The one-gluon-exchange interaction $U_{\rm OGE}$ is represented by diagram $U_{1,1}$. Here, an effective gluon is emitted and absorbed on different lines which causes a genuine interaction by the exchange of momentum. The same holds true for the effective two-gluon annihilation interaction $U_{\rm TGA}$ correspoding to diagram $U_{1,0}$: a $q\bar{q}$-pair of the same flavor is scattered into an other pair with different momenta. The one-gluon-annihililation interaction is absent in QCD, because a single gluon is colored and cannot couple to the color-neutral sector. One should emphasize that the effective interaction as obtained with the method of iterated resolvents [5] has no operators which do not belong to one of the three classes of Eq.(1).

The structure of Eq.(1) has drastic consequences whenever one considers a realistic case for more than 1 flavor. – Why is that? Suppose, I was technically able to solve the effective Hamiltonian in Eq.(1) for just 1 flavor as a matrix diagonalization problem, as a warm-up exercise. Suppose, I want to treat next the case for 3 flavors. The matrix dimension increases by a factor $3 \times 3 \sim 10$, and the numerical effort for diagonalization on a computer increases thus by a factor 1000. For the physical 6 flavors the effort is correspondingly larger.

But the symmetries in the effective Hamiltonian of Eq.(1) are quite helpful, as demonstrated in Fig.2. The matrix shown in this figure visualizes the kernel of the effective Hamiltonian as a matrix of block matrices [4]. Each block matrix represents a contribution from $H_{\rm LC,eff}$. The symbol (E_i) stands for contributions from $T + U_{\rm OGE}$. Most of the blocks are zero block-matrices, *i.e.* all matrix elements inside a block vanish. For example, the block $u\bar{d} \leftrightarrow u\bar{s}$ vanishes because the Hamiltonian in Eq.(1) cannot connect them: The kinetic energy cannot connect them since it is a diagonal operator; the one-gluon-exchange interaction $U_{\rm OGE}$ cannot connect them, since the $\bar{d}$ cannot change suddenly into an s-anti-quark as seen from diagram $U_{1,1}$; and, finally, the two-gluon-annihilation cannot connect them since diagram $U_{1,0}$ requires the same flavor on the left (and/or on the right). The latter feature lets vanish also blocks like $u\bar{d} \leftrightarrow s\bar{s}$. This demonstrates that most of the Hamiltonian

is reducible and that one can diagonalize blockwise. Thus, only block matrix sectors like $u\bar{u} \leftrightarrow s\bar{s}$ are non-zero due to the diagrams $U_{1,0}$ and cause a mixing of flavors, as consequence of QCD. They are denoted by A_i in the figure.

It is thus reasonable to introduce a one-gluon-exchange Hamiltonian and to diagonalize it on its own merit,

$$H_{\rm OGE}\,|\Psi_{f\bar{f}'}\rangle = (T + U_{\rm OGE})\,|\Psi_{f\bar{f}'}\rangle = M^2_{f\bar{f}'}\,|\Psi_{f\bar{f}'}\rangle, \tag{2}$$

to obtain flavor masses $M_{f\bar{f}'}$ and the associated wave functions $\Psi_{f\bar{f}'}$.

3 Formulation of the model

Diagonalizing $H_{\rm OGE}$ and generating the many eigenfunctions $\Psi_{f\bar{f}';i}$ can also be understood as the generation of a unitary transformation to pre-diagonalize the flavor mixing matrix. Although

$$\langle\Psi_{f\bar{f};i}|U_{\rm TGA}|\Psi_{f'\bar{f}';j}\rangle = 0, \qquad \text{for } i \neq j, \tag{3}$$

would be a false statement, in general, one can expect that the off-diagonal matrix elements $(i-j)$ are (much) smaller than those on the diagonal $(i-i)$. Requiring Eq.(3) to be true, however, makes things all of a sudden very simple: The huge flavor-mixing matrix reduces to a state-by-state diagonalization of a n_f by n_f flavor-mixing matrix $H_{\rm M}$, where n_f is the number of flavors. Eq.(3) will be refered to as model assumption I.

It is thus reasonable to introduce the ground-state-ground-state correlations

$$a_{ff'} \equiv \langle\Psi_{f\bar{f}}|U_{\rm TGA}|\Psi_{f'\bar{f}'}\rangle. \tag{4}$$

Since $m_u = m_d$, one has

$$a_{uu} = a_{dd} = a_{ud} = a_{dd} \equiv a, \qquad \text{and } M_{d\bar{d}} = M_{u\bar{u}}. \tag{5}$$

I introduce as model assumption II

$$a_{us} = a_{dc} = a_{ub} = \ldots = a_{bb} \equiv a, \tag{6}$$

just to reduce their number. In principle, the gs-gs correlations could be calculated from the wave functions, but below I will use a as an adjustable parameter. It can be different for pseudo-scalar and vector mesons.

Table 1
The calculated mass eigenvalues in MeV. Those for singlet-1s states are given in the lower, those for singlet-2s states in the upper triangle. Taken from [6].

	$\overline{u}$	$\overline{d}$	$\overline{s}$	$\overline{c}$	$\overline{b}$
u		768	871	2030	5418
d	140		871	2030	5418
s	494	494		2124	5510
c	1865	1865	1929		6580
b	5279	5279	5338	6114	

Table 2
The empirical masses of the flavor-off-diagonal physical mesons in MeV. The vector mesons are given in the upper, the scalar mesons in the lower triangle. [6].

	$\overline{u}$	$\overline{d}$	$\overline{s}$	$\overline{c}$	$\overline{b}$
u		768	892	2007	5325
d	140		896	2010	5325
s	494	498		2110	—
c	1865	1869	1969		—
b	5278	5279	5375	—	

Solutions to Eq.(2) are actually available [6], within the so-called $\uparrow\downarrow$-model. Its results are compiled in Table 1 and compared to the experimental masses in Table 2, taken from the particle data group [7]. These data do not yet include the topped mesons, which is the reason that the top quark is omitted here and below. The parameters of the model are the physical ones, the strong coupling constant α and the quark masses m_q. They are the same as in [6] and tabulated in Table 3. A similar model had been considered in earlier preliminary work [8].

For the present purpose, the codes for $\uparrow\downarrow$-model have been run again to get the eigenvalues of Eq.(2) for the flavor diagonal case with no new parameters to adjust. There are given in Tables 5 and 6.

4 Flavor SU(2) and SU(3)

Let us restrict first to 2 flavors with equal masses $m_u = m_d$. The flavor-mixing matrix reduces to a 2 by 2 matrix, with

$$H_{\mathrm{M}} = \begin{array}{c c} & \begin{array}{cc} u\bar{u} & \qquad d\bar{d} \end{array} \\ \begin{array}{c} u\bar{u} \\ d\bar{d} \end{array} & \begin{pmatrix} a + M_{u\bar{u}}^2 & a \\ a & a + M_{u\bar{u}}^2 \end{pmatrix} \end{array}. \tag{7}$$

The diagonalization of $H_{\mathrm{M}}|\Phi_i\rangle = M_i^2|\Phi_i\rangle$ is easy. The two eigenstates,

$$|\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} |u\bar{u}\rangle \\ -|d\bar{d}\rangle \end{pmatrix}, \ |\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} |u\bar{u}\rangle \\ |d\bar{d}\rangle \end{pmatrix}, \tag{8}$$

Table 3
Model parameters: $\alpha = 0.6904$, quark masses in MeV. [6].

q	m_q
u	406
d	406
s	508
c	1666
b	5054

Table 4
The wave function of physical neutral pseudo-scalar mesons in terms of the $q\bar{q}$-wave functions. The leading component is normalized to 10.

	π^0	η	η'	η_c	η_b
$u\overline{u}$	10.000	-9.313	5.360	0.310	0.031
$d\overline{d}$	-10.000	-9.313	5.360	0.310	0.031
$s\overline{s}$	-0.000	10.000	10.000	0.326	0.031
$c\overline{c}$	-0.000	0.251	-0.658	10.000	0.034
$b\overline{b}$	0.000	0.025	-0.061	-0.037	10.000

are associated with the eigenvalues

$$M_1^2 = M_{u\bar{u}}^2, \qquad M_2^2 = M_{u\bar{u}}^2 + 2a. \tag{9}$$

The assumption of equal quark masses leads thus to $M_{u\bar{d}} = M_{d\bar{u}} = M_1$. They can be arranged into a mass degenerate triplet of isospin 1, independend of the numerical value of a.

Next, consider 3 flavors. The flavor mixing matrix for the ground state becomes

$$H_{\rm M} = \begin{array}{c} \\ u\bar{u} \\ d\bar{d} \\ s\bar{s} \end{array} \!\!\begin{array}{c} \begin{array}{ccc} \quad u\bar{u} \quad & \quad d\bar{d} \quad & \quad s\bar{s} \quad \end{array} \\ \begin{pmatrix} a + M_{u\bar{u}}^2 & a & a_{us} \\ a & a + M_{u\bar{u}}^2 & a_{ds} \\ a_{us} & a_{ds} & a_{ss} + M_{s\bar{s}}^2 \end{pmatrix} \end{array}. \tag{10}$$

The model assumption Eq.(6) changes that into a matrix with elements

$$\langle f | H_{\rm M} | f' \rangle = a + M_{f\bar{f}}^2 \, \delta_{f,f'}. \tag{11}$$

If one assumes $m_u = m_d = m_c = m$, thus $M_{s\bar{s}} = M_{u\bar{u}}^2$, as above, $H_{\rm M}$ can be diagonalized againin closed form. The three eigenstates

$$|\Phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} |u\bar{u}\rangle \\ -|d\bar{d}\rangle \\ 0|s\bar{s}\rangle \end{pmatrix}, |\Phi_2\rangle = \frac{1}{\sqrt{6}} \begin{pmatrix} -|u\bar{u}\rangle \\ -|d\bar{d}\rangle \\ 2|s\bar{s}\rangle \end{pmatrix}, |\Phi_3\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} |u\bar{u}\rangle \\ |d\bar{d}\rangle \\ |s\bar{s}\rangle \end{pmatrix}, \tag{12}$$

are associated with the eigenvalues

$$M_1^2 = M_{u\bar{u}}^2, \qquad M_2^2 = M_{u\bar{u}}^2, \qquad M_3^2 = M_{u\bar{u}}^2 + 3a. \tag{13}$$

Table 5
Compilation for the neutral pseudo-scalar mesons with $a = (491\ \text{MeV})^2$. Masses are given in MeV.

	$M_{f\bar{f}}$	M	M_{exp}
π^0	140	140	135
η	140	485	549
η'	661	*958	958
η_c	2870	2915	2980
η_b	8922	8935	—

Table 6
Compilation for the neutral pseudo-vector mesons with $a = (255\ \text{MeV})^2$. Masses are given in MeV.

	$M_{f\bar{f}}$	M	M_{exp}
ρ^0	768	768	768
ω	768	832	782
Φ	973	*1019	1019
J/Ψ	3231	3242	3097
Υ	9822	9825	9460

The coherent state picks up all the strength, again. The eigenvalues of the remaining two states coincide with the unperturbated ones. State Φ_1 can again be interpreted as the eigenstate for the charge neutral π^0 and the mass of the coherent state Φ_3 could be fitted with the η'. But then state Φ_2 is degenerate with the π^0: Instead of a mass triplet one has a mass quadruplet.

Obviously, one cannot abstract from the appreciable mass difference between the up and strange quark. A 3×3 matrix like in Eq.(11) can not be diagonalized in a simple way. It was diagonalized analytically but approximately in [8].

5 Flavor SU(5), and its breaking by mass terms

Since one has to do numerical work anyway it is reasonable to proceed immediately to five flavors. The five flavor mixing-matrix H_{M} is given in Eq.(11), just with $n_f = 5$. The diagonal elements $M^2_{ff'}$ are obtained from Table 5. Diagonalizing $H_{\text{M}}|\Phi\rangle = M^2|\Phi\rangle$ numerically, produces the wavefunctions Φ in Table 4 and the physical masses M in Table 5. The parameter a is used to reproduce the mass of the η', as indicated by the star * in the table. The corresponding results for pseudo-vector mesons are found in Table 6.

By adjusting one single parameter, one reproduces three empirical facts: (1) the mass of the π^0 is (strictly) degenerate with $\pi^\pm$ (isospin); (2) the unperturbed mass of the η is lifted from the comparably small value of 140 MeV to the comparatively large value of 485 MeV; (3) the unperturbed mass of the η' is lifted by roughly 50% the meet the experimental value.

The wave functions have also remarkable properties, as seen from Table 4. The numerical results in the table remind to the SU(3) pattern in Eq.(12). Particularly the isospin-pattern of the π^0 and the coherent-state pattern of the η'-wavefunction should emphasized. The heavy quark admixtures are small.

6 Conclusion

In the present light-cone approach to gauge theory with an effective interaction isospin is not a dynamic symmetry, but a consequence of equal up and down mass. Flavor-SU(3) is an approximate symmetry. The approach explains even the phenomenological observation that flavor-SU(3) symmetry works better than SU(4) or SU(5); the large mass of the heavy mesons dominates the flavor-mixing matrix so strongly that the symmetry induced by the annihilation interaction is destroyed. The present work contributes to the η-η' puzzle [9] and exposes an accuracy comparable to state-of-art lattice gauge calculations [10]. To the best of my knowledge no other model including the phenomenological ones [1] covers the whole range of flavored hadrons with the same set of parameters.

The present approach is however in conflict with other theoretical constructs. Zero modes are absent since one works with the light-cone gauge $A^+ = 0$ [4]. In consequence there are no chiral condensates which seem to be so important otherwise. They are not needed here since the parameter a provides the additional mass scale. It will be calculated from the theory in future work, removing then all parameter dependence beyond α and m. At least one knows now that this is worth an effort.

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